Formula Sheet

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Definition 3.10

A random variable is said to have a hypergeometric probability distribution if and only if

where is an integer 0, 1, 2,..., , subject to the restrictions and .

Theorem 3.10

If is a random variable with a hypergeometric distribution

and

Poisson Probability Distribution Definition 3.11

A random variable is said to have a Poisson probability distribution if and only if

Theorem 3.11

If is a random variable possessing a Poisson distribution with parameter λ, then

and

By definition

Notice that the first term in this sum is equal to 0 (when = 0), and, hence,

As it stands, this quantity is not equal to the sum of the values of a probability function over all values of , but we can change it to the proper form by factoring out of the expression and letting − 1. Then the limits of summation become (when ) and(when ), and

.

Notice that is the probability function for a Poisson random variable, and . Therefore,. Thus, the mean of a Poisson random variable is the single parameterthat appears in the expression for the Poisson probability function. We leave the derivation of the variance as Exercise 3.138.

Definition 3.12

The is defined to be () and is denoted by

Definition 3.13

The or the *central moment of* is defined to be [(] and is denoted by

Definition 3.14

The moment-generating function for a *random variable* is defined to be . We say that a moment-generating function for exists if there exists a positive constant such that is finite for

Theorem 3.12

If exists, then for any positive integer ,

In other words, if you find the th derivative of with respect to and then set The result will be

Definition 3.15

Let be an integer-valued random variable for which = , where = 0, 1, 2, . . . . The probability-generating function for is defined to be

for all values of such that is finite.

Definition 3.16

The th *factorial moment* for a random variable is defined to be

where is a positive integer.

Theorem 3.13

If is the probability-generating function for an integer-valued random variable, , then the th factorial moment of is given by

Theorem 3.14

Let be a random variable with mean and finite variance . Then, for any constant > 0,

Definition 4.1

Let denote any random variable. The *distribution function* of , denoted by is such that for

Theorem 4.1

**Properties of a Distribution Function** If is a distribution function, then

1. is a nondecreasing function of y. [If is a nondecreasing function of . [If and are any values such that , then (

Definition 4.2

A random variable with distribution function is said to be *continuous* is continuous, for -

Definition 4.3

Let be the distribution function for a continuous random variable. Then given by

wherever the derivative exists, is called the *probability density function* for the random variable

Theorem 4.2

**Properties of a Density Function** If is a density function for a continuous random variable, then

1. for all y, .

Definition 4.4

Let denote any random variable. If the th quantile of , denoted by , is the smallest value such that ≥ . If is continuous, is the smallest value such that ) = ) = p. Some prefer to call the 100th percentile of.

Theorem 4.3

If the random variable has density function and , then the probability that falls in the interval is

Definition 4.5

The expected value of a continuous random variable is

provided that the integral exists.

Theorem 4.4

Let be a function of ; then the expected value of is given by

provided that the integral exists.

Theorem 4.5

Let be a constant and let be functions of a continuous random variable Then the following results hold:

Definition 4.6

If a random variableis said to have a continuous *uniform probability distribution* on the interval (, ) if and only if the density function of is

Definition 4.7

The constants that determine the specific form of a density function are called *parameters* of the density function.

Theorem 4.6

If and is a random variable uniformly distributed on the interval(, ), then

and .

Definition 4.8

A random variable is said to have a *normal probability distribution* if and only if, for and , the density function of is

Theorem 4.7

If is a normally distributed random variable with parameters and , then

Theorem 4.13

Tchebysheff’s Theorem Let Y be a random variable with finite mean and variance . Then, for any

Definition 5.1

Let and be discrete random variables. The *joint* (or bivariate) *probability* *function* for and is given by

Theorem 5.1

If and are discrete random variables with joint probability function then

1. for all
2. , where the sum is over all values that are assigned nonzero probabilities.

Definition 5.2

For any random variables and , the joint (bivariate) distribution function is

Definition 5.3

Let and be continuous random variables with joint distribution function If there exists a nonnegative function , such that

For all , then and are said to be *jointly continuous random variables.* The function is called the *joint probability density function.*

Theorem 5.2

If and are random variables with joint distribution function , then

1. .
2. .
3. If and then

((((.

Theorem 5.2

If and are jointly continuous random variables with a joint density function given by (, then

1. ( for all .
2. .

Definition 5.4

**a** Let and be jointly discrete random variables with probability function (. Then the *marginal probability functions* of and , respectively, are given by

**b** Let and be jointly continuous random variables with joint density function (. Then the *marginal density functions* of and , respectively, are given by

and .

Definition 5.5

If and are jointly discrete random variables with joint probability function and marginal probability functions and , respectively, then the *conditional discrete probability function* of given is

provided that .

Definition 5.6

If and are jointly continuous random variables with jointly density function , then the *conditional distribution function* of given is

.

Definition 5.7

Let and be jointly continuous random variables with joint density and marginal densities and , respectively. For any such that , the conditional density of given

= is given by

and, for any such that , the conditional density of given is given by

Definition 5.8

Let have distribution function , have distribution function (, and and

Have joint distribution function . Then and are said to be *independent* if and only if

=

for every pair of real numbers .

If and are not independent, they are said to be *dependent*.

Theorem 5.4

If and are discrete random variables with joint probability function and marginal probability functions and , respectively, then and are independent if and only if

for all pairs of real numbers .

If and are continuous random variables with joint probability function and marginal probability functions and , respectively, then and are independent if and only if

for all pairs of real numbers .

Theorem 5.5

Let and have a joint density that is positive if and only if and , for constants a, b, c, and d; and otherwise. Then and are independent random variables if and only if

Where is a nonnegative function of alone and is a nonnegative function of alone.